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Odd Integers N With Five Distinct Prime Factors for Which $2 - 10^{-12} < \sigma(N)/N < 2 + 10^{-12}$

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Abstract. We make a table of odd integers N with five distinct prime factors for which $2 - 10^{-12} < \sigma(N)/N < 2 + 10^{-12}$, and show that for such N $|\sigma(N)/N - 2| > 10^{-14}$. Using this inequality, we prove that there are no odd perfect numbers, no quasiperfect numbers and no odd almost perfect numbers with five distinct prime factors. We also make a table of odd primitive abundant numbers N with five distinct prime factors for which $2 < \sigma(N)/N < 2 + 2/10^{10}$.

1. A positive integer N is called perfect, quasiperfect (QP), or almost perfect according as $\sigma(N) = 2N$, $2N + 1$, or $2N - 1$, respectively, where $\sigma(N)$ is the sum of the positive divisors of N. While twenty-four even perfect numbers are known, no odd perfect (OP) numbers, no QP numbers, and no almost perfect numbers except a power of 2 are known.

In this paper we make a table of odd integers N with five distinct prime factors for which

$$2 - 10^{-12} < \sigma(N)/N < 2 + 10^{-12}, \tag{1}$$

and we show that for such N

$$|\sigma(N)/N - 2| > 10^{-14}.$$

Using this inequality, we prove that there are no OP, QP, or odd almost perfect (OAP) numbers with five distinct prime factors.

N is called primitive abundant if N is abundant ($\sigma(N) > 2N$) and every proper divisor M of N is deficient ($\sigma(M) < 2M$). In 1913 Dickson [4] published a table of odd primitive abundant numbers with less than five distinct prime factors. In this paper we also make a table of odd primitive abundant numbers N with five distinct prime factors for which

$$2 < \sigma(N)/N < 2 + 2/10^{10}. \tag{2}$$

2. Throughout this paper we let $N = \prod_{i=1}^{r} p_i^{a_i}$ where $3 \leqslant p_1 < \cdots < p_r$ are primes and a_i's are positive integers. $p_i^{a_i}$ is called a component of N.

We define

$$a(p) = \min\{a \mid p^{a+1} > 10^{12}\},$$
$$\omega(N) = r,$$
$$S(N) = \sigma(N)/N = \prod_{i=1}^{r} (p_i^{a_i+1} - 1)/p_i^{a_i}(p_i - 1),$$

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$$A(N) = \left[\prod_{a_i<a(p_i)} S(p_i^{a_i})\right]\left[\prod_{a_i\geq a(p_i)} S(p_i^{a(p_i)})\right],$$

$$B(N) = \left[\prod_{a_i<a(p_i)} S(p_i^{a_i})\right]\left[\prod_{a_i\geq a(p_i)} p_i/(p_i-1)\right],$$

$$L(p^a) = \begin{cases} [10^{12} \log S(p^a)]/10^{12} & \text{if } a < a(p), \\ [10^{12} \log p/(p-1)]/10^{12} & \text{if } a \geq a(p), \end{cases}$$

where [ ] is the greatest integer function. We note that if p, q are primes with $p > q$ and a, b are positive integers then

$$S(p^a) = (p^{a+1}-1)/p^a(p-1) < p/(p-1) = \lim_{a\to\infty} S(p^a) \leq (q+1)/q \leq S(q^b),$$

and so $L(p^a) \leq L(q^b)$ and $A(N) \leq S(N) \leq B(N)$. Hence, we have

LEMMA 1. (a) *If* $A(N) > 2 - 10^{-12}$ *and* $B(N) < 2 + 10^{-12}$, *N satisfies* (1).

(b) *If* $A(N) \leq 2 - 10^{-12} < B(N) < 2 + 10^{-12}$, *some N satisfies* (1).

(c) *If* $2 - 10^{-12} < A(N) < 2 + 10^{-12} \leq B(N)$, *some N satisfies* (1).

(d) *If* $A(N) < 2 - 10^{-12}$ *and* $2 + 10^{-12} < B(N)$, *some N may satisfy* (1).

(e) *If* $2 + 10^{-12} < A(N)$ *or* $B(N) < 2 - 10^{-12}$, *N does not satisfy* (1).

In Lemmas 2 through 5 we assume that N satisfies (1) and $\omega(N) = 5$.

LEMMA 2.

$$0.6931471805544 < \sum_{i=1}^{5} L(p_i^{b_i}) < 0.6931471805655, \tag{3}$$

where $b_i = \min\{a_i, a(p_i)\}$.

Proof. Suppose p^a is a component of N. If $a < a(p)$, then

$$|\log S(p^a) - L(p^a)| < 10^{-12}.$$

If $a \geq a(p)$, then $p^{a+1} > 10^{12}$ and

$$10^{-12} > \log p/(p-1) - L(p^a) > \log S(p^a) - L(p^a) \geq \log S(p^a) - \log p/(p-1)$$
$$= \log(1 - 1/p^{a+1}) = -\sum_{i=1}^{\infty} 1/i(p^{a+1})^i > -1/(p^{a+1}-1) \geq -10^{-12}.$$

Hence

$$|\log S(p^a) - L(p^a)| < 10^{-12}.$$

Since (1) holds,

$$0.6931471805544 < \log(2 - 10^{-12}) - 5/10^{12}$$
$$< \sum_{i=1}^{5} \log S(p_i^{a_i}) - 5/10^{12} < \sum_{i=1}^{5} L(p_i^{b_i})$$
$$< \sum_{i=1}^{5} \log S(p_i^{a_i}) + 5/10^{12} < \log(2 + 10^{-12}) + 5/10^{12}$$
$$< 0.6931471805655. \quad \text{Q.E.D.}$$

LEMMA 3. $p_1 = 3, p_2 \leqslant 11$ *and* $p_3 \leqslant 41$.

Proof. Lemma 3 follows from the following inequalities:

$$\frac{5}{4}\frac{7}{6}\frac{11}{10}\frac{13}{12}\frac{17}{16} < 2 - 10^{-12},$$

$$\frac{3}{2}\frac{13}{12}\frac{17}{16}\frac{19}{18}\frac{23}{22} < 2 - 10^{-12},$$

$$\frac{3}{2}\frac{5}{4}\frac{43}{42}\frac{47}{46}\frac{53}{52} < 2 - 10^{-12}. \quad \text{Q.E.D.}$$

LEMMA 4. $p_4 < 5000$.

Proof. Suppose N satisfies (1) and $p_4 \geqslant 5003$. Then

$$0 \leqslant L(p_5^{b_5}) \leqslant L(p_4^{b_4}) < \log S(p_4^{b_4}) + 10^{-12}$$
$$< \log p_4/(p_4 - 1) + 10^{-12} < 1/(p_4 - 1) + 10^{-12}$$
$$< 0.0002.$$

Hence by (3)

$$0.69274 < \sum_{i=1}^{3} L(p_i^{b_i}) < 0.69315. \tag{4}$$

A computer (PDP11 at the University of Toledo) was used to find $\Pi_{i=1}^{3} p_i^{b_i}$ satisfying (4), but there were none. Q.E.D.

Similarly, we can prove

LEMMA 5. $p_5 < 3000000$, *or* $\Pi_{i=1}^{4} p_i^{b_i} = 3^7 5^6 17^2 233$ *and* $36549767 \leqslant p_5 \leqslant 36551083$.

The computer was used to find $N = \Pi_{i=1}^{5} p_i^{a_i}$ satisfying $a_i \leqslant a(p_i)$, Lemmas 3, 4, 5, and Lemma 2 or Lemma 1(b), (c), (d), with the result given in Table 1.

LEMMA 6. *Suppose* $N = \Pi_{i=1}^{5} p_i^{a_i}$ *and* $M = \Pi_{i=1}^{5} p_i^{b_i}$ *where* $b_i = \min\{a_i, a(p_i)\}$.
If $M = 3^{23}5^{12}17^6 257^4 65521$, $|S(N) - 2| > 5/10^{13}$;
if $M = 3^8 5^{14} 17^3 251 \cdot 1884529$, $|S(N) - 2| > 2/10^{14}$;
if $M = 3^8 5^9 17^3 251 \cdot 1579769$, $|S(N) - 2| > 3/10^{13}$;
if $M = 3^8 5^8 17^9 269^4 4153^3$, $|S(N) - 2| > 4/10^{14}$;
if $\Pi_{i=1}^{4} p_i^{b_i} = 3^7 5^6 17^2 233$, $|S(N) - 2| > 10^{-14}$.
In all other cases $|S(N) - 2| > 10^{-13}$.

Proof. The first part of Lemma 6 follows from the following inequalities:

$$S(3^{23}5^{12}17^6 257^4 65521) < 2 - 5/10^{13},$$
$$S(3^{23}5^{12}17^6 257^5 65521) > 2 + 1/10^{12},$$
$$S(3^8 5^{14} 17^3 251)\ 1884529/1884528 < 2 - 2/10^{14},$$
$$S(3^8 5^9 17^3 251 \cdot 1579769) < 2 - 4/10^{13},$$
$$S(3^8 5^9 17^3 251 \cdot 1579769^2) > 2 + 3/10^{13},$$
$$S(3^8 5^8 269^4)\ 17/16 \cdot 4153/4152 < 2 - 4/10^{14},$$
$$S(3^8 5^8 17^9 269^5 4153^3) > 2 + 3/10^{13},$$
$$S(3^7 5^6 17^2 233 \cdot 36550379) > 2 + 5/10^{14},$$

and

$$S(3^7 5^6 17^2 233)\ 36550429/36550428 < 2 - 10^{-14}.$$

Suppose $|S(N) - 2| \leqslant 10^{-13}$. Then (1) holds, and so N is given in Table 1; however, for every N in Table 1 except for those given above $S(N) \leqslant B(N) < 2 - 10^{-13}$, or $S(N) \geqslant A(N) > 2 + 10^{-13}$. Q.E.D.

We have proved

THEOREM. *If N is an odd integer with $\omega(N) = 5$, $|\sigma(N)/N - 2| > 10^{-14}$.*

3. We used a similar method to find odd primitive abundant numbers $N = \Pi_{i=1}^{5} p_i^{a_i}$ for which (2) holds, with the result given in Table 2 in the microfiche. Table 2 includes odd primitive abundant numbers N with $\omega(N) = 5$ one of whose component p^a is greater than 10^{10}; for, letting $M = N/p^a$, we have

$$\begin{aligned} 2 < \sigma(N)/N = \sigma(M)\sigma(p^a)/Mp^a &= \sigma(M)(p\sigma(p^{a-1}) + 1)/Mp^a \\ = \sigma(Mp^{a-1})/Mp^{a-1} &+ \sigma(M)/Mp^a < 2 + 2/10^{10}, \end{aligned}$$

showing that (2) holds.

4. Suppose N is an odd integer such that $\sigma(N) = 2N + A$. If $|A/N| \leqslant 10^{-14}$, then by our Theorem $\omega(N) \geqslant 6$. We give three examples of such N.

Suppose N is OP. Sylvester (1888), Dickson (1913), and Kanold (1949) proved that $\omega(N) \geqslant 5$. From our Theorem we have

PROPOSITION 1. *If N is OP, $\omega(N) \geqslant 6$.*

This fact was also proved by Gradšteĭn (1925), Kühnel (1949) and Webber (1951). Pomerance [1] (1972) and Robbins (1972) proved that $\omega(N) \geqslant 7$, and Hagis [2] proved that $\omega(N) \geqslant 8$.

PROPOSITION 2. *If N is QP, $\omega(N) \geqslant 6$.*

Proof. By [3] if N is QP, then N is an odd perfect square, $\omega(N) \geqslant 5$ and $N > 10^{20}$. Hence $2 < S(N) = 2 + 1/N < 2 + 10^{-20}$, and so by Theorem $\omega(N) \geqslant 6$. Q.E.D.

LEMMA 7. *If N is OAP, pN is primitive abundant for some $p|N$.*

Proof. Suppose $N = \Pi_{i=1}^{r} p_i^{a_i}$ is OAP, and choose j so that $\sigma(p_j^{a_j}) \geqslant \sigma(p_i^{a_i})$ for every i. Letting $p = p_j$, $a = a_j$ and $L = N/p^a$, we have

$$\begin{aligned} 2p^aL - 1 &= \sigma(N) = \sigma(p^a)\sigma(L) \\ &= (1 + p\sigma(p^{a-1}))\sigma(L) = \sigma(L) + p\sigma(p^{a-1})\sigma(L). \end{aligned}$$

Hence $p|\sigma(L) + 1$. If $p = \sigma(L) + 1$, then

$$\begin{aligned} \sum_{i=1}^{a+1} p^i &= \sigma(p^a)p = \sigma(p^a)\sigma(L) + \sigma(p^a) \\ &= \sigma(N) + \sigma(p^a) = 2p^aL - 1 + \sigma(p^a) = 2p^aL + \sum_{i=1}^{a} p^i, \end{aligned}$$

or $p^{a+1} = 2p^aL$, showing that $N = 2^a$. Since N is OAP, $p \neq \sigma(L) + 1$, and so $p < \sigma(L)$ because $p|\sigma(L) + 1$. Then

$$\begin{aligned} \sigma(pN) &= \sigma(p^{a+1})\sigma(L) = (1 + p\sigma(p^a))\sigma(L) \\ &= \sigma(L) + p\sigma(N) = \sigma(L) + 2pN - p > 2pN, \end{aligned}$$

showing that pN is abundant.

Suppose M is a proper divisor of pN. If $p^{a+1} \nmid M$, then M is a divisor of N, and M is deficient because

$$S(M) \leqslant S(N) = 2 - 1/N < 2.$$

Suppose $p^{a+1} | M$. Then for some k, $p_k^{a_k} \nmid M$. Letting $q = p_k$ and $b = a_k$, we have $\sigma(p^a) \geqslant \sigma(q^b)$, or

$$\sum_{i=1}^{b} q^i \leqslant \sum_{i=1}^{a} p^i < \sum_{i=1}^{a+1} p^i.$$

Hence

$$(1/p^{a+1}) \sum_{i=0}^{b-1} q^{-i} < (1/q^b) \sum_{i=0}^{a} p^{-i},$$

and by adding $\Sigma_{i=0}^{a} p^{-i} \, \Sigma_{i=0}^{b-1} q^{-i}$ to both sides we obtain

$$\sum_{i=0}^{a+1} p^{-i} \sum_{i=0}^{b-1} q^{-i} < \sum_{i=0}^{a} p^{-i} \sum_{i=0}^{b} q^{-i},$$

or $S(p^{a+1})S(q^{b-1}) < S(p^a)S(q^b)$. Then

$$S(M) \leqslant S(p^{a+1})S(q^{b-1}) \prod_{i \neq j,k} S(p_i^{a_i})$$

$$< S(p^a)S(q^b) \prod_{i \neq j,k} S(p_i^{a_i}) = S(N) < 2,$$

showing that M is deficient. Q.E.D.

LEMMA 8. *If* $N = \Pi_{i=1}^{r} p_i^{a_i}$ *is OAP,* a_i *is even. If* $p_1 = 3, a_1 \geqslant 12$.

Proof. Suppose N is OAP, p^a is a component of N, q is a prime and $q | \sigma(p^a)$. Since $\sigma(N) = 2N - 1$ is odd and $\sigma(p^a) | \sigma(N)$, $\sigma(p^a) = \Sigma_{j=0}^{a} p^j$ is odd. Hence a is even. Since $q | 2\sigma(N) = 4N - 2$ and $4N$ is a perfect square, $(2|q) = 1$, where $(2|q)$ is the Legendre symbol, and so $q \equiv 1$ or $7 \pmod 8$ because $(2|q) = (-1)^{(q^2-1)/8}$. Also $\sigma(p^a) \equiv 1$ or $7 \pmod 8$, for, otherwise, $\sigma(p^a)$ would have a prime factor $\equiv 3$ or $5 \pmod 8$.

Suppose $p = 3$ and $a = 2e$. Then $\sigma(3^{2e}) \equiv 1 + 4e \equiv 1$ or $7 \pmod 8$, or $e \equiv 0 \pmod 2$. Hence $a = 4, 8, 12, \ldots$; however, $a \neq 4$ or 8 because $11 | \sigma(3^4)$, $11 \equiv 3 \pmod 8$, $13 | \sigma(3^8)$ and $13 \equiv 5 \pmod 8$. Q.E.D.

PROPOSITION 4. *If* N *is OAP,* $\omega(N) \geqslant 6$.

Proof. Suppose $N = \Pi_{i=1}^{r}$ is OAP. Then by Lemma 7 pN is primitive abundant for some $p | N$. If $3 \nmid N$, $\omega(N) \geqslant 7$, for, otherwise,

$$2 < S(pN) < \prod_{i=1}^{r} \frac{p_i}{p_i - 1} \leqslant \frac{5}{4}\frac{7}{6}\frac{11}{10}\frac{13}{12}\frac{17}{16}\frac{19}{18} < 2.$$

Suppose $3 | N$. Then $3^{12} | pN$ by Lemma 8. According to the table of odd primitive abundant numbers M with fewer than five distinct prime factors in [4] $3^{12} \nmid M$.

Hence $\omega(N) \geqslant 5$, and $N \geqslant 3^{12}5^2 7^2 11^2 13^2 > 10^{13}$. Then $2 > S(N) = 2 - 1/N > 2 - 10^{-13}$, and by Lemma 6 $\omega(N) \geqslant 6$. Q.E.D.

For other results on QP and OAP see [3], [5], [6], [7] and [8].

Computer time for Tables 1 and 2 was over four hours.

TABLE 1

$N = \Pi_{i=1}^5 p_i^{a_i}$ *for which* $2 - 10^{-12} < \sigma(N)/N < 2 + 10^{-12}$(a)

$p_1^{b_1}$	$p_2^{b_2}$	$p_3^{b_3}$	$p_4^{b_4}$	$p_5^{b_5}$
3^{25}	5^5	17^7	251	570407(b)
3^{23}	5^{12}	17^6	257^4	65521(c)
3^{22}	5^5	17^6	251	569659^2
3^{21}	5^9	17^9	257^4	65099^2(b)
	5^5	17^5	251	557273
3^{20}	5^{14}	17^5	257^4	65357(b)
3^{19}	5^3	17^3	181	57149^2
3^{18}	5^5	17^5	251	557017^2
		17^4	251	406811^2
3^{16}	5^5	17^8	251	567943^2
3^{12}	5^5	17^5	251	412943^2
3^{11}	5^{12}	17^9	257^3	58337(c)
3^{10}	5^{10}	17^9	257^3	47791^2(c)
3^9	7^3	13^5	19^2	1009643(b)
3^8	5^{16}	17^8	257^4	15137^2(c)
	5^{14}	17^3	251	1884527(c)
				1884529
	5^{13}	17^3	251	1884061(c)
	5^{11}	17^3	251	1870207
	5^9	17^3	251	1579769
	5^8	17^9	269^4	4153^3(d)
	5^3	19^9	83^6	493277
		19^8	83^3	488203^2
		19^7	83^4	493201
3^7	5^6	17^2	233	(e)

Note: (a) If $b_i = a(p_i)$ and $c > 0$, Np_i^c also satisfies (1). See Lemma 1(a).
(b) See Lemma 1(b). (c) See Lemma 1(c). (d) See Lemma 1(d).
(e) $36549767 \leqslant p_5 \leqslant 36551083$.

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TABLE 2

Odd primitive abundant numbers $N = \prod_{i=1}^{5} p_i^{a_i}$ for which $2 < \sigma(N)/N < 2 + 2/10^{10}$.

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$p_1^{a_1}$	$p_2^{p_2}$	$p_3^{p_3}$	$p_4^{p_4}$	$p_5^{p_5}$
3^{24}	5^{12}	17^{6}	257^{4}	65521*
	5^{5}	17^{6}	251	570407^{2*}
3^{23}	5^{12}	17^{6}	257^{5}	65521*
			257^{3}	65521^{2*}
	5^{5}	17^{6}	251	569659*
		17^{5}	251	557281*
		17^{4}	251	406951*
3^{22}	5^{12}	17^{6}	257^{2}	65269*
		17^{5}	257^{3}	65353*
	5^{10}	17^{8}	257^{4}	65447*
	5^{9}	17^{8}	257^{4}	65099^{2*}
	5^{7}	17^{4}	263	8191^{2*}
	5^{5}	17^{7}	251	570403*
		17^{6}	251	569659^{2*}
		17^{4}	251	406951^{2*}
	5^{3}	17^{5}	181	179651*
3^{21}	5^{14}	17^{5}	257^{4}	65357*
	5^{10}	17^{9}	257^{4}	65447*
		17^{8}	257^{3}	65447^{2*}
		17^{5}	257^{3}	65269*
		17^{4}	257^{4}	62563*
	5^{9}	17^{8}	257^{2}	64849*
		17^{3}	257^{3}	36583*
		17	137^{4}	18461^{2*}
	5^{8}	17^{5}	257^{4}	63241*
			257^{3}	63241^{2*}
	5^{7}	17^{6}	257^{2}	55927*
	5^{6}	17	137^{2}	14869^{2*}
	5^{5}	17^{8}	251	570421*
		17^{7}	251	570379* $\le p_5$*
				$\le$ 570391*
		17^{5}	251	557261* $\le p_5$*
				$\le$ 557273*

NOTE: * means one component of N is greater than 10^{10}.

	5^3	17^5	181	179651^{2*}
		23^3	47^5	5393*
	5^2	19^4	59^4	709*
3^{20}	5^{15}	17^5	257^4	65357*
	5^{14}	17^5	257^3	65357^2
	5^{12}	17^4	257^3	62639
	5^{11}	17^5	257^2	65089
		17^4	257^4	62627
			257^3	62627^2
		17^3	257^2	36637^2
	5^{10}	17^7	257^4	65447^2
		17^6	257^4	65437^2
		17^4	257^3	62563^2
	5^9	17^9	257^2	64849*
		17	137^5	18461^{2*}
	5^8	17^6	257^4	63397
			257^3	63397^2
	5^5	17^9	251	570421*
		17^8	251	$570389 \leq p_5$
				≤ 570419
		17^7	251	570359
				570373
		17^6	251	$569599 \leq p_5$
				≤ 569623
	5^3	23^3	47^6	5393*
3^{19}	5^{14}	17^5	257^4	65357^2
	5^{13}	17^3	257^3	36721^2
	5^{12}	17^4	257^4	62639
			257^3	62639^2
	5^{11}	17^5	257^2	65089^2
		17^4	257^4	62627^2
	5^{10}	17^4	257^4	62563^2
	5^9	17^8	257^2	64849^2
		17^3	257^4	36583
	5^8	17^5	257^4	63241^2
	5^7	17^6	257^2	55927^2
		19^5	101^2	1907^2

	5^5	17^9	251	570359^*
		17^8	251	570329
	5^4	17^2	227^3	44453
	5^3	17^6	181	180907
3^{18}	5^{14}	17^4	257^3	62639
	5^{13}	17^6	257^4	65521^2
			257^3	65519
			257^2	65269^2
		17^5	257^4	65353
			257^3	65353^2
	5^{12}	17^4	271^2	4591
	5^{11}	17^6	257	32887
	5^9	17^5	257^4	64919
	5^7	17^8	257^2	55933^2
		17^5	257^3	55987^2
		17^4	257^2	53813^2
		19^4	97^3	8969
	5^5	17^9	251	570173^*
		17^8	251	570139
				570161
		17^7	251	$570107 \le p_5$
				≤ 570113
		17^6	251	569369
		17^5	251	556987
				556999
		17^4	251	406807
				406811^{2*}
		17^2	239	164117
	5^4	17^2	227^3	44453^2
	5^3	17^5	181	179623
3^{17}	5^{14}	17^4	257^4	62633
	5^{12}	17^8	257^4	65521^2
			257^3	65519
	5^{11}	17^5	257	32843
		17^4	257^4	62617
		-	281^2	2861^{3*}
	5^{10}	17^8	257^4	65437^2
	5^8	17^9	257^4	63397^*
		17^8	257^3	63397^2

		17	137^2	18191
	5^7	17^7	257^2	55927^2
	5^5	17^8	251	$569581 \leq p_5$
				≤ 569609
		17^6	251	568807
				568823
		17^5	251	556459
				556477
				556483^{2*}
		17^4	251	406513
				406517
		17^2	239	164071
	5^4	17^2	227^2	44281
	5^3	17^7	181	180907
	7^9	13^2	19^2	44357
3^{16}	5^{15}	17^4	257^2	62383*
	5^{14}	17^5	257^3	65323
		17^4	257	32143^2
	5^{12}	17^8	257^3	65497
		17^7	383^2	769
		17^3	257^3	36709
	5^{11}	17^4	257^4	62597
			257^3	62597^2
		17^3	281	2677
	5^{10}	17^9	257^3	65413*
		17^7	257^4	65413
			257^3	65413^2
		17^4	257^4	62533
			257^3	62533^2
	5^9	17^7	257^2	64817^2
		17^5	257^4	64891^2
		17^4	257^2	61987
	5^8	17^8	257^4	63377^2
		17^5	257^4	63211^2
	5^5	17^9	251	567943*
		17^8	251	567937
				567943^{2*}

		17^7	251	567871 $\le p_5$
				$\le$ 567883
				567899^{2*}
		17^6	251	567143
		17^5	251	554887
		17^4	251	405659
				405667
	5^3	17^6	181	180667^{2*}
3^{15}	5^{14}	17^7	257^2	65183
		17^5	257^3	65257
	5^{13}	17^8	257^2	65183
		17^6	257^3	65423
			257^2	65173
		17^4	257^2	62323^2
	5^{12}	17^6	257^2	65171
			383^3	769
		17^4	257^4	62549
	5^{11}	17^7	257^2	65167^2
		17^6	257^4	65407
	5^{10}	17^5	257^4	65171
			257^3	65171^2
		17^3	257^2	36583
	5^9	17^3	257	23531^2
	5^8	17^8	257^3	63313
		17^7	257^2	63079^2
		17^4	257^4	60611
	5^7	17^6	257^3	56039^2
		17^5	257^2	55733
	5^6	17^4	257^2	34667
	5^5	17^8	251	562963 $\le p_5$
				$\le$562987
		17^7	251	562931
				562943
		17^6	251	562193
				562201
		17^5	251	550139
		17^4	251	403133
				403141

		17^2	239	163517
	5^3	17^9	181	180241*
		17^8	181	180239
				180241^{2*}
		17^6	181	180161
		17^5	181	178903
		17^4	181	159937
		17^3	181	57073
3^{14}	5^{15}	17^3	257	23561*
	5^{13}	17^4	257^2	62143^2
	5^{12}	17^3	349^2	947^3
		19^7	97^4	9209
		19^3	97	4493
	5^{11}	17^7	257^2	64969
		17^7	257^2	64901^2
	5^8	17^5	257^2	62731
	5^7	17^9	257^4	55901*
		17^8	257^3	55901^{2*}
		17^7	257^2	55717
		17^5	257^2	55589
	5^5	17^8	251	548623
				548629
		17^7	251	548579
				548591
		17^6	251	$547889 \le p_5$
				$\le$ 547909*
		17^5	251	$536423 \le p_5$
				≤ 536447
				536449^{2*}
		17^4	251	395719
	5^4	17^5	239^2	24239^2
	5^3	17^5	181	177431^{2*}
		17^4	181	158759^{2*}
		17^3	181	56923^2
3^{13}	5^{14}	17^7	257^2	64403
		17^4	257^2	61609
	5^{13}	17^8	257^2	64403

	5^{11}	17^{8}	257^{4}	64633^{2}
		17^{4}	257^{4}	61819^{2}
	5^{10}	17^{8}	247^{2}	64319
		17^{7}	293	1973^{2}
	5^{9}	17^{8}	257^{3}	64223
		17^{7}	257^{4}	64223
		17^{4}	257^{2}	61223
	5^{8}	17^{7}	257^{2}	62347
		17^{3}	439^{3}	607
	5^{7}	17^{8}	257^{3}	55469^{2}
	5^{6}	17^{5}	257^{3}	35323
	5^{5}	17^{8}	251	$509647 \leq p_5$
				≤ 509659
		17^{7}	251	509623
		17^{6}	251	509023
				509027
		17^{4}	251	499117
				499127
		17^{4}	251	375029
				375043^{2*}
		17^{3}	251	71761^{2}
	5^{4}	17^{2}	229^{3}	16301^{2}
	5^{3}	17^{5}	181	173149^{2*}
3^{12}	5^{15}	17^{4}	263^{3}	9461^{2*}
	5^{13}	17^{4}	263^{4}	9461^{2}
	5^{12}	17^{5}	257^{2}	62549
	5^{10}	17^{5}	257^{2}	62573^{2}
		17^{3}	263	7607
	5^{8}	17^{7}	257^{2}	60763
		17^{5}	257^{2}	60611
		17^{4}	257^{2}	58271^{2}
	5^{5}	17^{7}	251	420097
				420103
		17^{6}	251	419687
				419693
		17^{5}	251	412939
				412943^{2*}
		17^{4}	251	324199

	5^4	17^4	239	16691
	5^3	17^5	181	161459^{2*}
		17^3	181	55171
3^{11}	5^{14}	17^5	257^3	58199
	5^{12}	17^9	257^3	58337*
		17^7	257^4	58337
		17^5	257	30937
	5^{10}	17^8	257^3	58271^2
	5^9	17^5	257	30841^2
	5^8	17^6	257^2	56453^2
	5^7	17^6	257^4	50753
		17^5	257^2	50503^2
	5^5	17^6	251	274943
		17^4	251	230467
		17^2	239	125407
	5^3	17^5	181	134263
3^{10}	5^{15}	17^7	257^4	47837*
	5^{14}	17^8	257	27743
		17^5	257^3	47743^2
		17^4	257^4	46279^2
	5^{13}	17^7	257^3	47837^2
		19^7	97^2	8677^2
	5^{12}	17^8	257^2	47701
	5^{10}	17^9	257^3	47791^{2*}
		17^7	257^4	47791^2
			257^2	47657^2
		17	137	8849^2
	5^9	17^6	257^3	47599^2
	5^6	17^4	257^3	29063^2
	5^5	17^5	251	134417
3^9	5^9	19	89^5	509
	5^7	17^6	257^4	28771
	5^5	17^8	251^4	355193
		17^7	251^4	355171
			251^3	355139
		17^6	251^4	354881
				354883
			251^3	354847

			251^2	347099
			251	53503^2
		17^5	251^4	350039
		17^4	251^4	284117
			251^3	284093
	5^3	17^6	181	44519^2
		17^3	181^4	255839
				255841
			181^2	245257
				245261^{2*}
	7^3	13^{10}	19^2	1276687*
		13^9	19^2	1276543* $\leq p_5$*
				$\leq$ 1276679 *
		13^8	19^2	1276397 $\leq$
				$\leq$ 1276529
		13^7	19^2	1274549 $\leq p_5$
				$\leq$ 1274671
		13^6	19^2	1251083 $\leq p_5$
				$\leq$ 1251227
		13^5	19^2	1009559 $\leq p_5$
				$\leq$ 1009637
3^8	5^{16}	17^8	257^4	15137^{2*}
	5^{15}	17^9	257^4	15137^{2*}
		17^3	251	1884529* $\leq p_5$*
				$\leq$ 1884611 *
	5^{14}	17^3	251	1884193 $\leq p_5$
				$\leq$ 1884527
	5^{13}	17^3	251	1883731 $\leq p_5$
				$\leq$ 1884061
	5^{12}	17^7	271^3	3733
		17^3	251	1881389 $\leq p_5$
				$\leq$ 1881697
	5^{11}	17^3	251	1869859 $\leq p_5$
				$\leq$ 1870207
	5^{10}	17^3	251	1814279 $\leq p_5$
				$\leq$ 1814599
	5^9	17^3	251	1579541 $\leq p_5$
				$\leq$ 1579751

			1579769^{2*}
	17^2	241^3	96517
5^8	17^9	269^5	4153^3
	17^3	251	$959083 \leq p_5$
			≤ 959131
	17^2	241^4	92849
		241^2	92237
5^7	17^6	283^3	2339
	17^4	257	11839
5^6	17^8	251	736577
	17^7	251	736511
	17^6	251	$735239 \leq p_5$
			≤ 735283
	17^5	251	$714751 \leq p_5$
			≤ 714787
	17^4	251	484987
5^4	17^9	233	477259*
	17^7	233	477209
			477221
	17^6	233	476683
			476701
	17^5	233	468001
5^3	19^9	83^6	493277*
		83^5	493277^{2*}
	19^8	83^6	493277^{2*}
		83^4	493211*
		83^3	488197*
			488203^{2*}
	19^7	83^5	493243
			493249
		83^4	$493177 \leq p_5$
			≤ 493201
		83^3	488171
		83^2	264811
	19^6	83^5	493001
		83^3	487933
		83^2	264739
	19^5	83^5	$488143 \leq p_5$
			≤ 488153

			83^3	483167
				483179
		19^4	83^4	411287
			83^3	$407783 \leq p_5$
				≤ 407791
			83^2	239231
				239233^{2*}
	5^2	17^8	83	48341^2
		19^2	53	125791
3^7	5^{14}	17^6	251^4	13457
	5^6	17^2	233	$36483767 \leq p_5$
				≤ 36550379
		37	47^4	61^4
	5^5	17^3	241	$662551 \leq p_5$
				≤ 662591
	5^4	17^7	229^3	99961
			229^2	99139
		17^5	229^3	99551^2
		17^4	229^2	92671^2
		17^3	227	365017
			229^3	45503^2
	5^3	17	109	$750413 \leq p_5$
				≤ 750457
	7^{12}	13^2	19^5	34267*
	7^3	13^4	19^3	$1193683 \leq p_5$
				≤ 1193783
3^6	5^{14}	17^6	229	71693^2
		17^3	229^4	145987
	5^{13}	17^3	229^4	145987^{2*}
	5^{12}	17^6	233^3	14251^2
		17^3	229^3	145963
	5^{11}	17^8	233	11287
		17^3	229^4	145903^{2*}
	5^{10}	17^5	229	71389^2
		17^3	229^3	145547^{2*}
	5^9	17^6	229	71171^2
		17^3	229^3	143831

	5^8	17^3	229^3	135829
		19^5	103^3	853^2
	5^6	17^7	229^4	130259
				130261^{2*}
			229^3	130253
		17^4	229^3	119311
		17^3	227	$2420849 \le p_5$
				≤ 2421421
	5^5	17^4	227^2	47563^2
	5^3	17^8	167	$422701 \le p_5$
				≤ 422711
		17^7	167	422689
		17^6	167	422267
		17^5	167	415427
		17^4	167	325729
		17^2	163	44983^2
	5	11^9	113^2	617
3^5	5^{16}	17^9	191^3	26861*
	5^{11}	19^2	83	73523^2
	5^8	17^7	191^4	26497
		17^6	193^4	10837^2
		23^2	47^6	27823^{2*}
	5^7	17^8	193^4	10601
		17^4	191^4	24697^2
		17^3	191^3	19319
	5^6	17^5	191^4	19973^2
	5^5	17^2	181^4	71597^2
		23^6	47^4	57991
	5^4	17^7	179^3	228311
			179^2	219619
		17^5	179^4	226231
			179^3	226183^{2*}
			179^2	217643
		17^4	179^4	196727
		17^3	179^2	60509
3^4	5^4	19^2	67^2	37993
3^2	5^4	11^7	137	62633
	5^2	11^3	67^4	31981